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| --- | --- | --- |
| https://lh4.googleusercontent.com/SPvkLsQz-5KaF-7uqXwqhrjwmbNxZ4qCCZAnhg3-Q024TrYF0A_9l6QyNZ5Zvzu3aH8itLneRVEpMIqsV1S89vn2YxwuiAoTc1TlDXvGevlf1TAttRmsGETDwmeqShyvP3EjOaiE | **PES University, Bengaluru**  (Established under Karnataka Act No. 16 of 2013) | **UE20CS904** |
| **March 2022: END SEMESTER ASSESSMENT (ESA)**  **M TECH DATA SCIENCE AND MACHINE LEARNING\_ SEMESTER I**  **UE20CS904 - Mathematical Foundation** | | |
| Time: 3 Hrs | Answer All Questions | Max Marks: 80 |
| **Instructions**   * All answers should be handwritten in the answer script. * Graph, if any has to be plotted in the graph sheet page of the answer script. * Marks will be allotted only when all the steps to arrive at the answer are shown. | | |
| **Section A (20 marks)** | | |

|  |  |  |  |
| --- | --- | --- | --- |
| 1 | a) | Are the column vectors for the matrix below independent? If they are not, which column vectors should we remove to make it independent? | 2 |
| b) | Find the vector projection of the vector  on | 2 |
| c) | In the plot shown below the dark shaded portion represents the original coordinates of an object and the same after transformation is represented by the lightly shaded object. Write the coordinates, the transformation matrix and the coordinates after transformation.  https://lh5.googleusercontent.com/xUVrZ26cKgJMvCe7exeIWEneR1cy12gPKg6qhNE2ZcqbQzNZtycN1AhpGZA68f5sZ__NUvNvE-lijNtuXWiN-2oV6CYPkuMU8i02oS5cmbV-UtnYmpRWVKTb2-LI_vf6ArWs32Uh | 2 |
| d) | Write the transformation matrix for clockwise rotation of a 2d image. | 2 |
|  | e) | State true or false:  (i) A singular value decomposition exists only for square matrix.  (ii) if is orthogonal | 2 |
|  | | | |
| 2 | a) | Check if the following are true or false:  (BA + A) = (B + I)A, where I is the identity matrix.  (A + B)C = AC + BC | 2 |
| b) | **What will happen when eigenvalues are roughly equal?**  A. PCA will perform outstandingly B. PCA will perform badly C. Can’t Say  D. None of above | 2 |
|  | c) | State the necessary condition for convex and concave function | 2 |
|  | d) | Find the critical points of the function | 2 |
|  | e) | Find the Jacobian Matrix of | 2 |
| **Section B (30 marks)** | | | |
| 3 | a) | Check whether the vectors  are linearly independent. and find the rank of the matrix. (3 marks)  (ii) Check it this matrix is orthogonal. (2 marks) | 5 |
| b) | Find Eigen Values of and for | 5 |
|  | c) | Find the eigenvectors of | 5 |
|  | d) | If . Calculate | 5 |
|  | e | Mr X is an investor. His portfolio primarily tracks the performance of the Nifty index and he wants to add the stock of company ‘A’. Before adding the stock to his portfolio, he wants to assess if there exists a relationship between Nifty and Stock A.  Year Nifty Stock A  2015 1692 68  2016 1978 102  2017 1884 110  2018 2151 112  2019 2519 154  Help Mr X to assess the same. | 5 |
|  | f | Obtain an orthonormal basis for  by applying Gram-Schmidt to the linearly independent set | 5 |
|  |  | **SECTION C – (3 \* 10 = 30 MARKS)** |  |
| 4 | a | For the image matrix on the left, use the kernel on the right and perform a convolution operation so as to replace the no ‘18’ with appropriate value.  Write your observation if this operation will increase or decrease the intensity values. (5 marks)    One convolution    Decompose the following matrix using SVD (5 Marks) | 10 |
| 4 | b | Consider a firm operating two plants in two different locations. They both produce the same output (say, 10 units) using the same type of inputs. Although the amounts of inputs vary between the plants the output level is the same.   1. The firm management suspects that the production cost in Plant 2 is higher than in Plant 1. Verify? 2. The manager of the Plant 2 claims that the reason of the cost differences is the higher input prices in her region than in the other. 3. Is the available information supports her claim?   The following information was collected from the managers of these plants. | 10 |
| 4 | c | Consider the data given below and fit a linear regression line using gradient descent.   |  |  |  |  |  | | --- | --- | --- | --- | --- | |  |  |  | 0.6 | 1 | |  |  |  | 0.48 | 0.95 |   Initialize the weights a and b to 0.8, 0.2 respectively. Update the weights such that the error is minimum using gradient descent. Use the function sum of squared errorswhere is the y-predicted value and is the actual given y. Plot the linear regression line after updating the values of a and b in two iterations. | 5 + 5 |